# AN EXPOSITORY NOTE ON THE ALTERNATIVE GOALS OF THE STATE OR PRINCIPAL

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#### Abstract

The aims of this study are i) to match possible theoretical goals with economic policy goals which are required for the development strategy and ii) to show the quantities required in order to reach those goals. This paper also assumes that the goals cannot be necessarily profit maximization, sales maximization and social welfare maximization, by itself. It can also be a hybrid combination of them. Based on this assumption quantity of production is shown for each of the combinations.

**Keywords** Profit maximization; Sales maximization; Social welfare maximization, State, Development.

# 1. Introduction

This study tries to give an answer to the question that what will i) the goal function and ii) the quantity of production of a country be in order to reach his objectives of development strategy?

Theoretically, we can suppose that there can be three objectives of development strategy: i) profit maximization, ii) total sales maximization, iii) consumer's surplus maximization. We want to match these *theoretical* objectives with the following *economic policy* objectives: i) capital accumulation, ii) increasing market share of national firms, iii) harmony of i and ii. Thus, this matching will give a framework about a country's i) goal function, ii) quantity of production.

Note that, each of those objectives which are pointed out above cannot be an objective by itself, necessarily. To us, assuming a hybrid objective function is more reasonable than assuming an objective function by itself. As an example, White (2001) assumes competing firms as private firms and a public firm, while public firm maximizes convex combination of consumer's surplus and social welfare, rather than profit. Similarly, White (2002) assumes that public firms maximize combination of producer's surplus and consumer's surplus. Mukherjee and Suetrong (2009) assume that the firm which makes foreign direct investment tries to maximize a convex combination of profit and social welfare. Similarly, Bös (1987) states that after privatization of a public firm the objective function will be a hybrid combination of profit and social welfare. Fershtman and Judd (1985), based on a principal agent model which includes an owner (principal) and two managers (agents), show that managers maximize a hybrid combination of profit and total sales. Thus, following that intuition in the literature we assume that firms can have an objective function which includes three convex combinations of possible goals: i) maximization of the combination of profit and sales, ii) maximization of the combination of profit and consumer's surplus, iii) maximization of the combination of social welfare (sum of producer's surplus or profit and consumer's surplus) and sales.

The study is organized as follows: Following section gives a diagrammatic explanation of alternative economic policy objectives and makes matching between theoretical objectives and economic policy objectives. Third section shows the quantity of production when the objective is maximization of the combination of profit and sales. Third section also shows the quantity of production when the objective is maximization of the combination of profit and sales. Third section also shows the quantity of production when the objective is maximization of the combination of profit and consumer's surplus, and the combination of social welfare and sales, respectively. Fourth section shows the same quantities for a multiple-plant monopoly. Finally, the study is concluded.

#### 2. Diagrammatic Explanation of the Alternative Economic Policy Objectives

One can obviously claim that profit is maximized when the objective is profit maximization and this will be matched with the capital accumulation objective.

On the other hand, we cannot easily claim that when the total sales are maximized, increasing national firms market share objective is achieved. Since this objective can also be achieved when social welfare is maximized which means that perfect competition conditions.

Figures 1, 2 and 3 give diagrammatic explanation of latter situation. For simplicity it is assumed that marginal cost is constant. Figures are drawn differently with respect to the intersection point of marginal cost and demand. In Figure 1, this intersection point is A and this means that the quantity of perfect competition is smaller than the quantity of the total sales maximization. However, Figure 2 shows that intersection point is B and this points out that the quantity of perfect competition is higher than the quantity of the

total sales maximization. Finally Figure 3 emphasizes that the quantities are equal for the perfect competition and total sales maximization.



Price, Marginal Revenue (MR), Marginal Cost (MC)



Source: Author's own.







Source: Author's own.







This means that increasing national firms' market share objective can be achieved when total sales are maximized (Figure 1) or when social welfare is maximized (perfect competition) (Figure 2) or when both of them are valid (Figure 3). Let us assume the existence of Figure 2 since it is reasonable to admit that for many textbooks quantity of production is maximum at perfect competition equilibrium. Then Table 1 will be valid.

Theoretical Objective	Quantity Produced (Q)	Welfare Cost (WC)	Level of Profit $(\pi)$	Economic Policy Objective
Profit $(\pi)$ Maximization	$Q_{\pi}$	WC <sub>P</sub>	$\pi_P$	Capital accumulation
Total Sales ( <i>TR</i> ) Maximization	$Q_{TR}$	WC <sub>TR</sub>	$\pi_{\scriptscriptstyle TR}$	Harmony between objectives of capital accumulation and increasing market share of national firms
Social Welfare (SW) Maximization	$Q_{sw}$	$WC_{SW} = 0$	$\pi_{_{SW}}=0$	Increasing market share of national firms

## Table 1.

Source: Author's own.

Thus according to the Table 1 and Figure 2, it will be  $Q_{\pi} < Q_{TR} < Q_{SW}$ ;  $WC_P > WC_{TR} > WC_{SW} = 0$ ;  $\pi_P > \pi_{TR} > \pi_{SW} = 0$ .

# 3. Quantity of Production under Alternative Hybrid Objectives

In order to begin to the analysis, we assume that there is a principal (the State) and two agents (firms). The State imposes to these firms to reach maximization of a hybrid combination of alternative objectives: i) profit maximization and sales maximization, ii) profit maximization and consumer's surplus maximization, iii) social welfare maximization and sales maximization.

Then, suppose that as a principal the State asks a question: *If these firms were in competition with each other, which quantity would they produce?* Assume that, the State can give an answer that question and calculate that quantity. Then he gives incentives to firms in order to reach that quantity, so the objective of his development strategy.

# 3.1. Maximization of the Combination of Profit and Sales

There are two firms (i = 1,2). The goal function for  $i^{\text{th}}$  producer  $(G_i)$  is a hybrid combination of profit and total sales:

$$G_i = \beta_i \pi_i + (1 - \beta_i) P q_i \tag{1}$$

where  $\beta$  is a parameter which represents a weight that the State imposes to the firm for profit  $(\pi)$  maximization. Then  $1 - \beta$  is the weight that the State imposes to the firm for total revenue maximization.

Profit equals to difference between total revenue (TR) and total cost (TC):

$$\pi_i = TR_i - TC_i \tag{2}$$

Assume that inverse demand function is the following:

$$P = a + bQ \tag{3}$$

where *a* and *b* are parameters and b < 0. *Q* is total quantity demanded at the market.

Then it will be:

$$\pi_i = Pq_i - j_i q_i \tag{4}$$

where *j* is average cost.

Then, (1) becomes (5):

$$G_i = \beta_i (a + bQ - j_i) q_i + (1 - \beta_i) (a + bQ) q_i$$
(5)

In order to obtain response function for the first firm let's write  $Q = q_1 + q_2$  in (5). It becomes:

$$G_{1} = \beta_{1}q_{1}a + \beta_{1}q_{1}^{2}b + \beta_{1}q_{1}q_{2}b - \beta_{1}j_{1}q_{1} + q_{1}a - q_{1}\beta_{1}a + q_{1}^{2}b + q_{1}bq_{2} - q_{1}^{2}\beta_{1}b - q_{1}\beta_{1}bq_{2}$$
(6)

Then, according to the first order conditions followings are written for maximization:

$$\frac{dG_1}{dq_1} = \beta_1 a + 2\beta_1 q_1 b + \beta_1 q_2 b - \beta_1 j_1 + a - \beta_1 a + 2q_1 b + bq_2 - 2\beta_1 q_1 b - \beta_1 q_2 b = 0$$
(7)

$$q_1 = \frac{\beta_1 j_1 - a - bq_2}{2b} \tag{8}$$

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Note that, we simply neglect to report the second order conditions.

It can be written for the second firm:

$$q_2 = \frac{\beta_2 j_2 - a - b q_1}{2b}$$
(9)

Using (8) and (9) following is written:

$$q_1 = \frac{2\beta_1 j_1 - a - \beta_2 j_2}{3b}$$
(10)

Likewise, (11) can be written:

$$q_2 = \frac{2\beta_2 j_2 - a - \beta_1 j_1}{3b}$$
(11)

#### 3.2. Maximization of the Combination of Profit and Consumer's Surplus

Now, the goal function for  $i^{\text{th}}$  producer  $(G_i)$  is a hybrid combination of profit and consumer's surplus  $(C^s)$ :

$$G_i = \alpha_i \pi_i + (1 - \alpha_i) C_i^s \tag{12}$$

where  $\alpha$  is a parameter which represents a weight that the State imposes to the firm for profit maximization. Then  $1-\alpha$  is the weight that the State imposes to the firm for consumer's surplus maximization.

When *i*<sup>th</sup> producer's production is  $q_i$  then its demand price will be  $P = a + bq_i$ (b < 0). The maximum price that the consumer can pay will be a. Thus, consumer's surplus can be calculated as follows:

$$C^{s} = \frac{a - (a + bq_i)}{2}q_i \tag{13}$$

$$C^{s} = \frac{-b q_{i}^{2}}{2}$$
(14)

Let's rewrite goal function:

$$G_{i} = \alpha_{i} \left[ (a + bQ) q_{i} - j_{i} q_{i} \right] + \left( 1 - \alpha_{i} \right) \left( \frac{-b q_{i}^{2}}{2} \right)$$
(15)

$$G_{1} = \alpha_{1}a + \alpha_{1}bq_{1}^{2} + \alpha_{1}bq_{1}q_{2} - \alpha_{1}j_{1}q_{1} - \frac{(1 - \alpha_{1})bq_{1}^{2}}{2}$$
(16)

Then, followings are written for maximization:

$$\frac{\partial G_1}{\partial q_1} = 2\alpha_1 b q_1 + \alpha_1 b q_2 - \alpha_1 j_1 - (1 - \alpha_1) b q_1 \tag{17}$$

$$\frac{\delta G_1}{\delta q_1} = 0 \tag{18}$$

$$2\alpha_1 bq_1 + \alpha_1 bq_2 - \alpha_1 j_1 - (1 - \alpha_1) bq_1 = 0$$
<sup>(19)</sup>

$$q_1 = \frac{-q_2\alpha_1 b + \alpha_1 j_1}{3\alpha_1 b - b}$$
(20)

Similarly, following can be obtained:

$$q_2 = \frac{-q_1\alpha_2 b + \alpha_2 j_2}{3\alpha_2 b - b} \tag{21}$$

Using (20) and (21) followings are written:

$$q_{1} = \frac{-\left(\frac{-q_{1}\alpha_{2}b + \alpha_{2}j_{2}}{3\alpha_{2}b - b}\right)\alpha_{1}b + \alpha_{1}j_{1}}{3\alpha_{1}b - b}$$
(22)

$$q_{1} = \frac{-\alpha_{2}\alpha_{1}j_{2} + \alpha_{1}j_{1}(3\alpha_{2} - 1)}{\left[b(3\alpha_{1} - 1)^{2} - \alpha_{2}\alpha_{1}b\right]}$$
(23)

Similarly, following can be written:

$$q_{2} = \frac{-\alpha_{1}\alpha_{2}j_{1} + \alpha_{2}j_{2}(3\alpha_{1} - 1)}{\left[b(3\alpha_{2} - 1)^{2} - \alpha_{1}\alpha_{2}b\right]}$$
(24)

## 3.3. Maximization of the Combination of Social Welfare and Sales

Finally, the goal function for  $i^{\text{th}}$  producer  $(G_i)$  is a hybrid combination of social welfare  $(S^w)$  and sales:

$$G_i = \beta_i S_i^{w} + (1 - \beta_i) P q_i$$
<sup>(25)</sup>

where  $\beta$  is a parameter which represents a weight that the State imposes to the firm for social welfare maximization. Then  $1-\beta$  is the weight that the State imposes to the firm for sales maximization.

Social welfare is equal to total surplus or sum of consumer's surplus and profit:

$$S^{W} = \frac{-b q_{i}^{2}}{2} + (a + bQ)q_{i} - j_{i}q_{i}$$
(26)

Let's rewrite goal function:

$$G_{i} = \beta_{i} \left( \frac{-b q_{i}^{2}}{2} + (a + bQ)q_{i} - j_{i}q_{i} \right) + (1 - \beta_{i})Pq_{i}$$
(27)

Since  $Q = q_1 + q_2$  and  $P = a + bQ = a + b(q_1 + q_2)$  (28) can be written:

$$G_{1} = \beta_{1} \left( \frac{b q_{1}^{2}}{2} + aq_{1} + bq_{1}q_{2} - j_{1}q_{1} \right) + (1 - \beta_{1})q_{1}a + (1 - \beta_{1})q_{1}^{2}b + (1 - \beta_{1})q_{1}q_{2}b$$

(28)

Then, followings are written for maximization:

$$\frac{\partial G_1}{\partial q_1} = \beta_1 b q_1 + \beta_1 a + \beta_1 b q_2 - \beta_1 j_1 + (1 - \beta_1) a + 2(1 - \beta_1) q_1 b + (1 - \beta_1) q_2 b$$
(29)

$$\frac{\partial G_1}{\partial q_1} = 0 \tag{30}$$

$$\beta_1 b q_1 + \beta_1 a + \beta_1 b q_2 - \beta_1 j_1 + (1 - \beta_1) a + 2(1 - \beta_1) q_1 b + (1 - \beta_1) q_2 b = 0$$
(31)

$$q_{1} = \frac{-q_{2}b + \beta_{1}j_{1} - a}{2b - \beta_{1}b}$$
(32)

Similarly, following can be obtained:

$$q_2 = \frac{-q_1 b + \beta_2 j_2 - a}{2b - \beta_2 b}$$
(33)

Using (32) and (33) followings are written:

$$q_{1} = \frac{-\left(\frac{-q_{1}b + \beta_{2}j_{2} - a}{2b - \beta_{2}b}\right)b + \beta_{1}j_{1} - a}{2b - \beta_{1}b}$$
(34)

.

$$q_{1} = \frac{\frac{-\beta_{2}j_{2} + a}{2 - \beta_{2}} + \beta_{1}j_{1} - a}{b\left[2 - \beta_{1} - \frac{1}{2 - \beta_{2}}\right]}$$
(35)

Similarly, following can be written:

$$q_{2} = \frac{\frac{-\beta_{1}j_{1}+a}{2-\beta_{1}} + \beta_{2}j_{2}-a}{b\left[2-\beta_{2}-\frac{1}{2-\beta_{1}}\right]}$$
(36)

# 4. Quantity of Production under Alternative Hybrid Objectives for a Multiple-Plant Monopoly

# 4.1. Maximization of the Combination of Profit and Sales

Suppose that the state is a principal and also owner of a multiple-plant monopoly. There are two plants of multiple-plant monopoly. Inverse demand function is given as:

$$P = a + bQ$$

where *P* is price, *Q* is quantity, *a* and *b* are parameters and b < 0.

There are two agents,  $i_1$  and  $i_2$  who are the managers of each plant. Assume that the objective function (*G*) of the multiple-plant monopoly covers a convex combination of profit and total revenue:

$$G = \alpha \pi + (1 - \alpha) P Q \tag{37}$$

where  $\alpha$  is a weight parameter which shows that owner of the multiple-plant monopoly imposes to the manager for profit  $(\pi)$  maximization. Then  $1-\alpha$  is the weight that owner of the multiple-plant monopoly imposes to the manager for total sales maximization.

Multiple-plant monopoly's profit will be:

$$\pi = \pi_1 + \pi_2 \tag{38}$$

Objective function is rewritten:

$$G = \alpha (\pi_1 + \pi_2) + (1 - \alpha) PQ \tag{39}$$

Profit is written as follows:

$$\pi = TR - TC \tag{40}$$

$$\pi = PQ - jQ \tag{41}$$

where j is average cost, TR is total sales and TC is total cost.

Using inverse demand function (41) can be rewritten:

$$\pi = (a + bQ)Q - jQ \tag{42}$$

Each plant's profit will be:

$$\pi_1 = (a + b(q_1 + q_2))q_1 - j_1q_1 \tag{43}$$

$$\pi_2 = (a + b(q_1 + q_2))q_2 - j_2q_2 \tag{44}$$

Total sales of multiple-plant monopoly will be:

$$TR = (a + b(q_1 + q_2))(q_1 + q_2)$$
(45)

Therefore (39) can be rewritten:

$$G = \alpha(((a + b(q_1 + q_2))q_1 - j_1q_1) + ((a + b(q_1 + q_2))q_2 - j_2q_2)) + (1 - \alpha)(a + b(q_1 + q_2))(q_1 + q_2)$$
(46)

When the goal function is maximized  $\left(\frac{\delta G}{\delta q_1}=0\right)$  and making rearrangements, it will be:

$$q_1 = \frac{\alpha j_1 - a}{2b} - q_2 \tag{47}$$

$$q_2 = \frac{\alpha j_2 - a}{2b} - q_1 \tag{48}$$

If (48) is put in (47) then (49) is written:

$$0 = \alpha \left(\frac{j_1 - j_2}{2b}\right) \tag{49}$$

(49) means that when average costs of plants are not equal to each other,  $\alpha$  should be zero; i.e. when average cost of plants is not same multiple-plant monopoly should only maximize its total sales.

#### 4.2. Maximization of the Combination of Profit and Consumer's Surplus

Suppose that the objective function of the multiple-plant monopoly is a convex combination of profit and consumer's surplus  $(C^s)$ :

$$G = \alpha \pi + (1 - \alpha)C^{s} \tag{50}$$

where  $\alpha$  is a parameter which includes a weight that state imposes to the plant for profit maximization. Therefore  $1-\alpha$  is the weight that state imposes to the plant for consumer's surplus maximization.

Each plant's profit will be:

$$\pi_1 = (a + b(q_1 + q_2))q_1 - j_1q_1 \tag{51}$$

$$\pi_2 = (a + b(q_1 + q_2))q_2 - j_2q_2 \tag{52}$$

Since production of *i*<sup>th</sup> producer is  $q_i$  then demand price of it will be  $P = a + bq_i$  (b < 0). As the maximum price that the consumer can pay will be a, then consumer's surplus will be:

$$C^{s} = \frac{-b q_{i}^{2}}{2}$$
(53)

For multiple-plant monopoly (53) will be:

$$C^{s} = \frac{-b q_{1}^{2}}{2} + \frac{-b q_{2}^{2}}{2}$$
(54)

Thus (50) can be rewritten as follows:

$$G = \alpha \left( \left( \left( a + b(q_1 + q_2) \right) q_1 - j_1 q_1 \right) + \left( \left( a + b(q_1 + q_2) \right) q_2 - j_2 q_2 \right) \right) + \left( 1 - \alpha \left( \frac{-b q_1^2}{2} + \frac{-b q_2^2}{2} \right) \right)$$
(55)

If the objective function is maximized  $\left(\frac{\delta G}{\delta q_1}=0\right)$  and leaving alone  $q_1$ , following can be

written:

$$q_1 = \frac{q_2(-2\alpha b) - \alpha a + \alpha j_1}{3\alpha b - b}$$
(56)

Similarly, (57) can be written:

$$q_2 = \frac{q_1(-2\alpha b) - \alpha a + \alpha j_2}{3\alpha b - b}$$
(57)

If we put the last equation in (56) then it will be:

$$q_{1} = \frac{\frac{2\alpha^{2}a - 2\alpha^{2}j_{2}}{3\alpha - 1} - \alpha a + \alpha j_{1}}{\left(3\alpha b - b - \frac{4\alpha^{2}b}{3\alpha - 1}\right)}$$
(58)

Similarly, (59) can be written:

$$q_{2} = \frac{\frac{2\alpha^{2}a - 2\alpha^{2}j_{1}}{3\alpha - 1} - \alpha a + \alpha j_{2}}{\left(3\alpha b - b - \frac{4\alpha^{2}b}{3\alpha - 1}\right)}$$
(59)

### 4.3. Maximization of the Combination of Social Welfare and Sales

Assume that the goal function is a hybrid combination of social welfare  $(S^w)$  and sales:

$$G = \beta S^{w} + (1 - \beta) PQ \tag{60}$$

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where  $\beta$  is a parameter which represents a weight that state imposes to the plant for social welfare maximization. Then  $1-\beta$  is the weight that state imposes to the plant for sales maximization.

Total profit equals to:

$$\pi = ((a + b(q_1 + q_2))q_1 - j_1q_1) + ((a + b(q_1 + q_2))q_2 - j_2q_2)$$
(61)

Consumer's surplus equals to:

$$C^{s} = \frac{-b q_{1}^{2}}{2} + \frac{-b q_{2}^{2}}{2}$$
(62)

Lastly, total sales equal to:

$$PQ = (a + b(q_1 + q_2))(q_1 + q_2)$$
(63)

Using (61), (62) and (63) objective function can be written as follows:

$$G = \beta \left( \left( \left( a + b(q_1 + q_2) \right) q_1 - j_1 q_1 \right) + \left( \left( a + b(q_1 + q_2) \right) q_2 - j_2 q_2 \right) + \frac{-b q_1^2}{2} + \frac{-b q_2^2}{2} \right) + \left( 1 - \beta \right) \left( \left( a + b(q_1 + q_2) \right) (q_1 + q_2) \right) \right)$$

$$(64)$$

Maximization of the objective function  $\left(\frac{\delta G}{\delta q_1}=0\right)$  gives the following results:

$$q_1 = \frac{\beta j_1 - a - 2bq_2}{\left(2b - b\beta\right)} \tag{65}$$

$$q_2 = \frac{\beta j_2 - a + q_1(-2b)}{(2b - b\beta)}$$
(66)

Finally, if we put the last equation in (65) then it will be:

$$q_{1} = \frac{\beta j_{1} - a - 2\left(\frac{\beta j_{2} - a}{2 - \beta}\right)}{2b - b\beta - \frac{4b}{2 - \beta}}$$
(67)

Likewise, (68) will be written:

$$q_{2} = \frac{\beta j_{2} - a - 2\left(\frac{\beta j_{1} - a}{2 - \beta}\right)}{2b - b\beta - \frac{4b}{2 - \beta}}$$

$$(68)$$

### **5.** Conclusions

This expository note's first aim is to match theoretical objectives with the economic policy objectives. According to our findings, assuming the conditions in Figure 2, if a country has an objective of i) capital accumulation then he targets profit maximization, ii) increasing market share of national firms then he targets social welfare maximization, iii) a harmony between capital accumulation and increasing market share of national firms then he targets total sales maximization.

Secondly, this expository note also shows the quantity of production for the alternative hybrid objectives as in (10), (11), (23), (24), (35), (36), (58), (59), (67), (68).

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